$$\therefore \quad \Sigma_1^n \frac{(n+2)^2}{n(n+4)} = n\left[1+\frac{1}{n+1}+\frac{1}{2(n+2)}+\frac{1}{3(n+3)}+\frac{1}{4(n+4)}\right].$$

Also solved by *Professor G. B. M. Zerr.*

35. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

Prove that the product of two numbers, each the sum of four (4) squares may be expressed as the sum of four squares in 48 different ways and unite some or all of the 48 ways.

Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $(a^2+b^2+c^2+d^2)(e^2+f^2+g^2+h^2)=A=$ the product of the two numbers. From Euler's Theorem we get

$$A=(ae+bf-cg-dh)^2+(af-be-ch+dg)^2$$
$$+(-ag+bh-ce+df)^2+(ah+bg+cf+de)^2,$$

$$=(-ae+bf-cg+dh)^2+(af+be+ch+dg)^2$$
$$+(ag+bh-ce-df)^2+(ah-bg-cf+de)^2,$$

$$=(ae-bf-cg+dh)^2+(af+be-ch-dg)^2$$
$$+(ag+bh+ce+df)^2+(-ah+bg-cf+de)^2,$$

$$=(ae+bf+cg+dh)^2+(-af+be-ch+dg)^2$$
$$+(ag-bh-ce+df)^2+(ah+bg-cf-de)^2,$$

$$=(ae-bf+cg+dh)^2+(af+be-ch+dg)^2$$
$$+(ag-bh-ce-df)^2+(ah+bg+cf-de)^2,$$

$$=(ae-bf-cg-dh)^2+(af+be+ch-dg)^2$$
$$+(ag-bh+ce+df)^2+(ah+bg-cf+de)^2,$$

$$=(ae+bf-cg+dh)^2+(af-be+ch+dg)^2$$
$$+(ag+bh+ce-df)^2+(ah-bg-cf-de)^2,$$

$$=(ae+bf+cg-dh)^2+(af-be-ch-dg)^2$$
$$+(ag+bh-ce+df)^2+(ah-bg+cf+de)^2.$$

The sum of four squares in eight different ways by combination of signs.

$$A=(ae+bf)-cg-dh)^2+(af-be-ch+dg)^2$$
$$+(-ag+bh-ce+df)^2+(ah+bg+cf+de)^2,$$

$$=(ag+bf-ch-de)^2+(af-bg-ce+dh)^2$$
$$+(-ah+be-cg+df)^2+(ae+bh+cf+dg)^2,$$

$$=(ah+bf-ce-dg)^2+(af-bh-cg+de)^2$$
$$+(-ae+bg-ch+df)^2+(ag+be+cf+dh)^2,$$
$$=(ae+bg-cf-dh)^2+(ag-be-ch+df)^2$$
$$+(-af+bh-ce+dg)^2+(ah+bf+cg+de)^2,$$
$$=(af+bg-ch-de)^2+(ag-bf-ce+dh)^2$$
$$+(-ah+be-cf+dg)^2+(ae+bh+cg+df)^2,$$
$$=(ah+bg-ce-df)^2+(ag-bh-cf+de)^2$$
$$+(-af+be-cg+dh)^2+(ae+bf+ch+dg)^2,$$

the sum of four squares in six different ways by combination of letters.

Since the signs of each of these six can form the sum of four squares in eight different ways, the whole number of ways is $8\times6=48$ different ways.

PROBLEMS.

46. Proposed by Professor WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics in the Ohio University, Athens, Ohio.

Find θ from $\cos\theta+\cos3\theta+\cos5\theta=0$.

47. Proposed by LEONARD E. DICKSON, A. M., Fellow in Mathematics, University of Chicago, Chicago, Illinois.

Prove that $(-1)(-1)=+1$.

GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

34. Proposed by T. JOHN COLE, Columbus, Ohio.

A circular field contains 10 acres. A horse is tied to the fence with a rope sufficiently long to graze over one acre. Find length of the rope (1) when the horse is on the inside (2) when he is on the outside of the fence.

Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.